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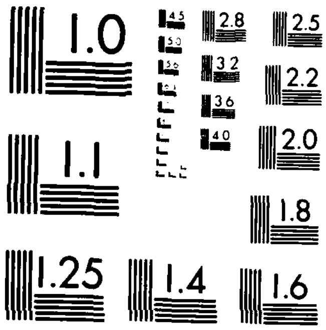

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A NOTE ON SOME SIMPLE INFLUENCE DIAGNOSTICS IN ACCELERATED LIFE TESTING

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A Note on Some Simple
Influence Diagnostics
in Accelerated Life
Testing

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ABSTRACT

Three methods for assessing influence in an accelerated life-testing model are considered; two different one-step approximations to the estimated parameter after case deletion and a method which treats extreme values at each design point as censored. These methods are compared using an example. Problems which occur when all observations at a design point are censored are discussed.

Key Words: Censored data, Linear regression, Regression diagnostics, EM algorithm.

1. INTRODUCTION

The focus of this paper is on understanding and assessing the influence of individual and groups of points on outcomes of interest in accelerated life testing. We will consider only a simple specific situation which occurs sufficiently often to merit study.

We shall assume that failure times have a distribution which can be described by unknown parameters, known environmental variables such as temperature, voltage, etc. and a distribution function which belongs to the class of accelerated failure time models, see Lawless (1983) and Kalbfleisch and Prentice (1980). Ordinarily, one will be concerned with the distribution at a specified set of environmental variables given by a vector x_o. However, experimenting at x_o may not prove to be practical because of time constraints. One solution is to run experiments at more extreme environmental conditions where failures can be expected to occur during the course of the experiment, and then to extrapolate the resulting model to the environment x of interest.

These kinds of experiments are often characterized by three components. The first is Type I censoring, i.e., the experiments are censored from above at fixed times. The second point is that in many of these experiments only a few test conditions $x_1,\ldots,x_k$ are run, but there are replicates at each test condition. The third component of the problem is that often the real parameter of interest is a percentile of the failure distribution at the environment of interest.

If T_{ij} denotes the failure time of the ith component at the jth test condition, then our model is

$$Y_{ij} = \log(T_{ij}) = x_j^T\beta+\sigma e_{ij} \quad j=1,\ldots,k,\ i=1,\ldots,n_j, \tag{1}$$

where the distribution function F and density f of the (e_{ij}) is known, e.g., extreme value (Lawless and Singal(1980)) or normal (Nelson and Hahn(1973)), leading to Weibull or lognormal models respectively for the failure times. The censoring times (S_j) are often fixed and depend only on the test condition, so we observe

$$Z_{ij} = \text{minimum}(Y_{ij},S_j), \tag{2}$$

$$\delta_{ij} = \begin{cases} 1 \text{ if the } (i,j)\text{th observation fails} \\ 0 \text{ if the } (i,j)\text{th observation is censored.} \end{cases} \tag{3}$$

The parameter of major importance is the $\alpha 100$th percentile of the failure distribution at a specified test condition x_o:

$$H(\sigma,\beta,x_o,\alpha) = \exp(x_o^T\beta+\sigma z_\alpha), \tag{4}$$

where

$$F(z_\alpha) = \alpha. \tag{5}$$

In this paper, various ways of assessing the influence of individual and groups of points on estimates of the $\alpha 100$th percentile $H(\sigma,\beta,x_o,\alpha)$ will be discussed through use of an example. Some of the considerations that arise are very different from what one ordinarily encounters in the linear regression case because of censoring and replication.

2. EXAMPLE

Crawford's (1970) data set, which considers the failure times of electrical insulation of motorettes as a function of temperature, will serve as an example to illustrate the proposed methods of assessing influence in an accelerated life testing situation. Ten motorettes were tested at each of the temperatures 220^oC, 190^oC, 170^oC and 150^oC, with interest focusing on the median and 10th percentile of the distribution at 130^oC, which as is typical in accelerated life tests, was unobserved. The

Arhenius law was used to model the data:

$$Y_{ij} = \log T_{ij} = \beta_o + \beta_1/t_a + \sigma e_{ij}, \tag{6}$$

where t_a is the absolute temperature and the (e_{ij}) are assumed to be normally distributed with mean zero and variance one. The available data at 15, 16, 19, 21 and 33 months into the experiment are listed in Table 1 and the data at 33 months is plotted in Figure 1.

If we let $\theta=(\beta,\sigma)$ and $\Lambda(\theta)$ be the log-likelihood of the data, then

$$\Lambda(\theta) = \sum\sum_{ij} L_{ij}(\theta), \tag{7}$$

where

$$L_{ij}(\theta) = \delta_{ij}\log\left[f(u_{ij}(\theta))/\sigma\right] + $$

$$+ (1 - \delta_{ij})\log\left[1 - F(u_{ij}(\theta))\right] \tag{8}$$

and

$$u_{ij}(\theta)= (y_{ij}-x_j{}^T\beta)/\sigma. \tag{9}$$

If X is the design matrix, then the maximum likelihood estimators (MLE) of β and σ are computed using the expectation maximization (EM) algorithm (Dempster et. al., 1977, Aitkin, 1981). The expectation step requires the computation of

$$y_{ij}^{*} = \delta_{ij} y_{ij} + (1-\delta_{ij}) E\left[y_{ij} \middle| y_{ij} > S_j\right] \tag{10}$$

and

$$y_{ij}^{*2} = \delta_{ij} y_{ij}^{*2} + (1-\delta_{ij}) E\left[y_{ij}^{*2} \middle| y_{ij} > S_j\right] \tag{11}$$

and β and σ are computed in the maximization step as

$$\beta = (X^T X)^{-1} X^T y^{*} \tag{12}$$

$$\sigma^2 = (1/n)(y^{*} - X\beta)^T (y^{*} - X\beta). \tag{13}$$

For the normal case we obtain

$$E\left[y_{ij}^{*} \middle| y_{ij} > S_j\right] = x_j^T \beta + \sigma \phi(u_{sj})/(1-\Phi(u_{sj})) \tag{14}$$

and

$$E\left[y_{ij}^{*2} \middle| y_{ij} > S_j\right] = (x_j^T \beta)^2 + \sigma^2 + \sigma(S_j + x_j^T \beta)\phi(u_{sj})/(1-\Phi(u_{sj})) \tag{15}$$

where

$$u_{sj} = (S_j - x_j^T \beta)/\sigma. \tag{16}$$

Since y^{*} is a function of the estimate of β, the actual maximum likelihood estimate is found by iteration and thus often denoted as the IMLE. An alternative method for computing estimates of β and σ was suggested by Schmee and Hahn(1979). Their iterative least square method (ILS) differs in the computation of σ in that $E\left[y_{ij}^2 \mid y_{ij}>S_j\right]$ is replaced by $E\left[y_{ij} \mid y_{ij}>S_j\right]^2$.

The IMLE along with the confidence intervals for the median and the 10th percentile are listed in Table 2. Note that σ is bias corrected as suggested by Aitkin(1981) and Tiku (1980), i.e., σ is replaced by $\sigma\left[n_o/(n_o-1)\right]^{\frac{1}{2}}$, where $n_o = \sum\sum_{ij}\delta_{ij}$.

Table 2. Maximum likelihood estimates of β and σ, along with the estimated median and 10th percentile lifetimes with their associated confidence intervals at 130°c.

Month	β_o	β_1	σ	Median Lifetime	Confidence interval for Median Lifetime: Lower Limit	Upper Limit	10th percentile Lifetime	Confidence interval for 10th percentile Lifetime: Lower Limit	Upper Limit
15	-4.887	3.853	.3980	46.8	17.7	123.8	14.5	4.7	44.3
16	-6.010	4.306	.2757	46.7	24.7	88.4	20.7	9.6	44.7
19	-6.541	4.539	.2359	52.1	31.2	86.8	26.0	14.8	45.5
21	-6.934	4.725	.2405	61.0	36.8	101.0	30.0	17.2	52.3
33	-6.379	4.460	.2128	48.1	32.7	70.9	25.7	16.9	39.0

The 16 month data have been extensively analyzed, see Nelson and Hahn (1972,1973), Schmee and Hahn (1979), Aitkin (1981), Nelson (1982) and Kalbfleisch and Prentice (1980). We are using these data illustratively and note only in passing that the linearity and normality assumptions appear fairly reasonable, although the constant variance assumption is questionable. Nelson (1982) states "the two earliest failures at 190^{o}C appear early compar the other data. ... The experiment was reviewed seek a cause of the early failures, but none was found. Ana... ...s yield the same conclusions whether or not these failures are included."

3. EFFECTS OF INDIVIDUAL AND PAIRS OF POINTS

In the usual case, influence diagnostics have been concerned with the effects of single case-deletion on constructs such as estimated parameters, likelihoods or ensembles of predicted values, see Cook and Weisberg (1982). With the exception of ordinary linear regression where explicit formulae for parameter estimates exist, computation of case-deletion influence diagnostics can be extremely time-consuming; it is usually somewhat expensive to construct influence diagnostics based on deleting pairs of points. For example, suppose in the motorette data we want to compute the influence of individual and pairs of observations on the percentiles $H(\sigma,\beta,x_o,\alpha)$. Then 40 maximizations would be required to assess the influence of individual points, while 780 maximizations would be required for pairs of observations. Such extensive computations will likely prove impractical. One way to avoid this computational morass is to delete only single observations and not consider pairs of points; this approach is sometimes unsatisfactory because it increases the chance of being trapped by masking, as we shall illustrate later.

Detection of influential observations for the censored linear model can be quite different from that for ordinary linear regression since outliers in the censored model can become more consequential than in the uncensored model. In the latter case outliers can influence the estimator β directly, but the effect need not be significant, since it depends as well on the leverage of the outliers. In the censored case, however, this is different, because an influential point can affect the estimation in two ways. A point which has not only a small residual at each iteration but also a large leverage can affect the estimation of β, and may affect the estimation of the expected times (14) to a certain extent since these are a function of β. However, a point with a large residual and a small leverage can cause the estimate of σ to be large and will have its impact on the estimator β through the estimation of the expected times (14) for censored points.

The usual way out of the computational dilemma is to replace maximum likelihood estimation after case or pair deletion by one-step approximations starting from the full data maximum likelihood estimator. This is the approach taken by Cook and Wang (1983) and by Hall, Rogers and Pregibon (1982), although neither of these authors consider deleting pairs of observations. Letting $\hat{\theta}$ be the maximum likelihood estimator of (β,σ), the one-step approximate maximum likelihood estimator based on deleting the (i,j)th observation is

$$\hat{\theta}_{(i,j)}=\hat{\theta}-\{R_{(i,j)}(\hat{\theta})\}^{-1}G_{ij}(\hat{\theta}), \tag{17}$$

$$G_{ij}(\hat{\theta})=\partial L_j(\hat{\theta})/\partial\theta \tag{18}$$

and

$$R_{ij}(\hat{\theta})=-\partial^2/\partial\theta^2 \wedge_{(i,j)}(\hat{\theta}) \tag{19}$$

where $\wedge_{(i,j)}(\theta)$ is defined by (7) with the (i,j)th point removed. Other, computationally simple approximations to the maximum likelihood estimate exist and do not require iteration, see Persson and Rootzen (1978) and Schneider (1984a). The Persson and Rootzen estimator avoids the iterative steps required for the other methods by setting u_{ij} in equation (9) equal to $u_{ij}=\Phi^{-1}(n_{oj}/n_j)$ where $n_{oj}=n_j - n_{rj}$ and n_{rj} is the number of censored observations at condition j. With this restriction an explicit estimator for $x_j^T\beta$ and σ_j at each test condition is obtained. The estimated times are obtained by substituting $\Phi^{-1}(n_{oj}/n_j)$ into (14) to get

$$E\left[Y_{ij}^{*}\middle|Y_{ij}>S_j\right]=X_j^T\beta + \sigma\phi\left[\Phi^{-1}(n_{oj}/n_j)\right]n_j/n_{rj}. \tag{20}$$

Then the expected loglife times estimated by (20) are used in equations (12) and (13) to arrive at the final estimates of β and σ which we will call the restricted maximum likelihood estimates (RML). Simulation studies of Schneider (1984b) suggest that this approximation, which we will name the restricted estimator, has behavior very similar to that of the maximum lkelihood estimator.

The replication at each test condition typically found in accelerated life-testing problems provides another way of avoiding massive computation to detect influential single and pairs of points, without having to resort to approximations to the maximum likelihood estimator. Specifically, we have found that it is often satisfactory to consider only the extremes at each test condition. For example, we can delete the smallest single and pair of points at each test condition, and then repeat the exercise for the largest single and pair of points. If there are k test conditions, then this involves estimating $\theta=(\beta,\sigma)$ only 4k times, a number which can be small enough to allow computation of the maximum likelihood estimate rather than approximations to it. For example, in the motorette data, there are k=4 test conditions so that the iterative algorithm used to compute the maximum likelihood estimate need only be employed 16 times, well within the bounds of feasibility. While this idea is not the same as deleting all possible pairs of points, it is often sufficient. In practice, it is rare that pairs of points are deleted to check for influence, so our idea does provide a simple way of expanding upon what is usually done. The idea of deleting extremes at each test condition can be extended easily to triples, etc.

A final method that we employ is to consider again the extremes at each test condition, but rather than using deletion techniques we censor observations. For example, the largest failure time at a test condition would be censored, with similar censoring at pairs, triples, etc. For the smallest censoring times we replace deletion by left or Type II censoring, see Tiku (1975). Full maximum likelihood estimation is used after these successive censorings. If we only consider single and pairs of observations at the k test conditions, we again need only 4k iterative maximizations.

We now illustrate these ideas on the motorette data, focusing on the effects of early failures on the predicted median lifetime at 130°C. The results are given in Table 3. It is interesting to note that none of the case deletion or Type II censoring methods suggest any problems when one considers single points. In fact, there appears to be real evidence of masking, as seen by the large changes in prediction when deleting or Type II censoring the early failures at 190°C, which were earlier labelled by Nelson (1982) as suspect. Note that case deletion followed by our restricted estimator also leads to concern about the early failures at 190°C, but the one-step approximation after case deletion gives us no clue as to the effect of these points. It would seem from this example that case-deletion and then one-step maximum likelihood estimation is unsatisfactory as a general technique for accelerated life-tests. As an approximation used for computational convenience, the restricted estimator seems to be preferable to the one-step maximum likelihood estimator.

Table 3. Influence of single and pairs of points for the 16^{th} month data measured in percentage of deviation of the median at 130°C. N_0 is number of deleted or censored points at test condition j.

Censoring °C	N_o	RML	one-step MLE	Type II censoring	MLE
220	1	1.3%	-0.9%	0.0%	0.2%
220	2	1.3%	0.9%	0.0%	0.6%
190	1	-1.8%	2.0%	-8.1%	-8.4%
190	2	-17.0%	2.4%	-22.1%	-21.3%
170	1	6.0%	8.4%	0.8%	9.9%
170	2	13.0%	12.2%	1.3%	20.8%

The results at 170°C also suggest that no major problems occur when a single point is deleted or censored. However, when pairs of points are considered the RML, one-step MLE, and MLE lead to concern about the two early failures at 170°C but the method of Type II censoring fails to call attention to this pair of points. It appears from this example that the method of Type II censoring is most appropriate for detecting points which do not preserve th structure of the distribution at a design point. When the structure of the distribution is preserved the expected times estimated by equation (14) will differ little from the observed times so that the censoring has a small impact on the estimates of β and σ. This also points to the restricted estimator as a desirable computational alternative to the other methods for the Crawford data.

4. THE INFLUENCE OF TOTAL CENSORED TEST CONDITIONS

The Crawford data at 16 months exhibits a fairly common characteristic, namely that for one test condition, all the data points are censored. Most often, this will happen at the test condition nearest the enviroment x_o in which we have interest.

An analysis of the asymptotic variance reveals that the variance of the slope estimator can be strongly dependent upon whether or not there are any uncensored observations at the extreme design points. The variance of the estimator β depends, generally speaking, upon the method of censoring. An equal percentage of censored observations at all design points will primarily affect the bias and variance of the intercept rather than that of the slope. Unequal censoring at the design points, as is typical for accelerated life tests, will mainly increase the variance of the slope.

Table 4. Estimated asymptotic variance of IMLE β; four test conditions with ten replications; entries have to be multiplied by 10^{-1}.

Percent of censored observation	Equal censoring at each design point		Equal censoring but last test condition has p% observations censored			
			p=90%		p=100%	
	$V(\hat{\beta}_o)$	$V(\hat{\beta}_1)$	$V(\hat{\beta}_o)$	$V(\hat{\beta}_1)$	$V(\hat{\beta}_o)$	$V(\hat{\beta}_1)$
10	1.527	.203	1.840	.320	2.375	.509
20	1.573	.209	1.873	.327	2.664	.550
30	1.644	.217	1.924	.335	2.554	.543
40	1.748	.229	2.001	.346	2.712	.572
50	1.907	.244	2.123	.361	2.950	.611

Table 4 gives the asymptotic variance of the IMLE of $\hat{\beta}$ estimated from four test conditions ($x=1,2,3,4$; $\sigma=1$) where the first three conditions are subject to equal censoring and the last condition is subject to 90% or 100% censorship. We see, for instance, that if 10% of the observations are censored at each design point, the asymptotic variance of the intercept and the slope are $V(\hat{\beta}_o)=.1530$ and $V(\hat{\beta}_1)=.0204$. When, on the other hand, 10%

of the observations at the first three conditions and all observations at the last condition are censored, the asymptotic variances increase to $V(\hat{\beta}_o)=.2381$ and $V(\hat{\beta}_1)=.0509$. Thus, the loss of efficiency of β is more than 100%. We also notice that the variance of $\hat{\beta}_o$ is reduced significantly, namely to $V(\hat{\beta}_1)=.0316$, by simply adding one uncensored observation at the last design point. In other words, for this case, the variance of the slope estimator increases about 60% when the last remaining observation at the fourth design point is censored as well.

These results can become important for accelerated life tests. Returning to our example we conclude that prediction based on the 16th and 19th months is not as good as that based on the 21st and 33rd month data due to the total censoring at $150^{o}C$. As the asymptotic variance suggests, the estimators should improve when the first failure time $150^{o}C$ is added to the 16th month data. When this failure time is added we obtain a predicted median lifetime at $130^{o}C$ of 57,238 and a 95% confidence interval of (30895,106042) hours for the estimates based on the entire data set and 46,415 hours and a 95% confidence interval of (32136,67038) hours for the estimates when the two early failures at $190^{o}C$ are Type II censored. Indeed the estimates of the median lifetime at $130^{o}C$ only improve when the influence of the outliers is bounded.

In Figure 2 we plot the estimated median at 130°C and the associated confidence intervals for the complete data and with the two earliest failures at 190°C being censored; a similar figure for the 10th percentile is given in Figure 3. These figures show that the predictions stabilize after 21 months, i.e. when more precise information about the failures at 150°C is available.

5. CONCLUSIONS

The results presented here suggest that, for the purpose of assessing influence in the Crawford data set, the restricted estimator was preferable to the one-step maximum likelihood estimator, at least as a computationally simple approximation to the actural maximum likelihood estimator. Using Type II censoring as a method for assessing influence seems to be adequate for pointing out observations which cause the variance of the failure time at a particular design point to be different from the variance at the other design points. These methods also point out the problems which can occur when all observations at a design point are censored.

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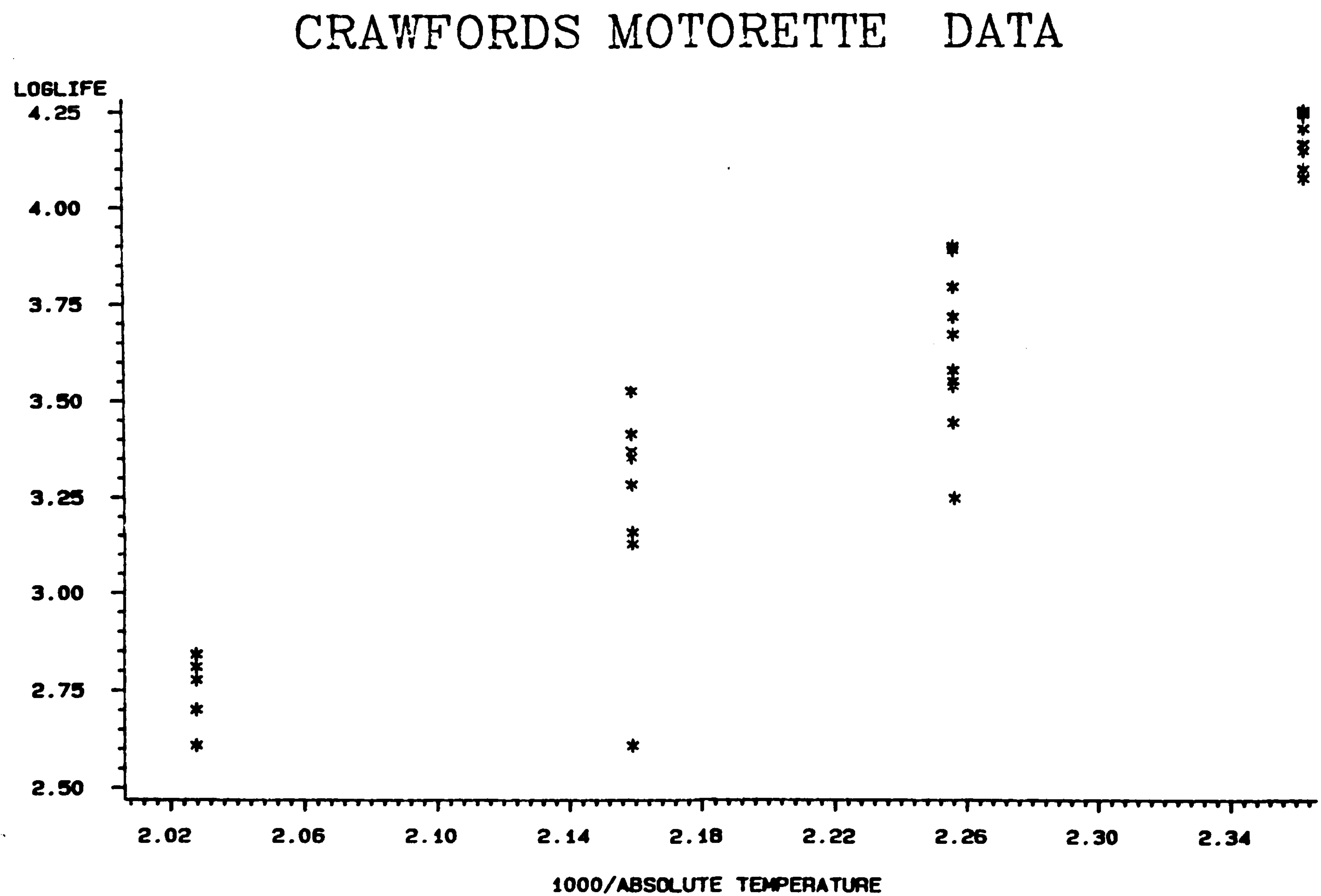

Table 1 - Plot of Crawford's Motorette Data

Table 1. Accelerated life test for 40 motorettes (Crawford, 1970).

Temperature °C	Lifetime (hours)	Censoring Times (hours) 15 months	16 months	19 months	21 months	33 months
220	408	336	528	700	700	17,661
220	408	336	528	700	700	17,661
220	504	336	528	700	700	17,661
220	504	336	528	700	700	17,661
220	504	336	528	700	700	17,661
220	600	336	528	700	700	17,661
220	600	336	528	700	700	17,661
220	648	336	528	700	700	17,661
220	648	336	528	700	700	17,661
220	696	366	528	700	700	17,661
190	408	1,296	1,680	3,120	4,000	17,661
190	408	1,296	1,680	3,120	4,000	17,661
190	1,344	1,296	1,680	3,120	4,000	17,661
190	1,344	1,296	1,680	3,120	4,000	17,661
190	1,440	1,296	1,680	3,120	4,000	17,661
190	1,920	1,296	1,680	3,120	4,000	17,661
190	2,256	1,296	1,680	3,120	4,000	17,661
190	2,352	1,296	1,680	3,120	4,000	17,661
190	2,596	1,296	1,680	3,120	4,000	17,661
190	3,360	1,296	1,680	3,120	4,000	17,661
170	1,764	5,112	5,448	6,792	7,632	17,661
170	2,772	5,112	5,448	6,792	7,632	17,661
170	3,444	5,112	5,448	6,792	7,632	17,661
170	3,542	5,112	5,448	6,792	7,632	17,661
170	3,780	5,112	5,448	6,792	7,632	17,661
170	4,680	5,112	5,448	6,792	7,632	17,661
170	5,196	5,112	5,448	6,792	7,632	17,661
170	6,206	5,112	5,448	6,792	7,632	17,661
170	7,716	5,112	5,448	6,792	7,632	17,661
170	7,884	5,112	5,448	6,792	7,632	17,661
150	11,781	7,392	8,064	9,429	11,421	17,661
150	12,453	7,392	8,064	9,429	11,421	17,661
150	13,897	7,392	8,064	9,429	11,421	17,661
150	14,469	7,392	8,064	9,429	11,421	17,661
150	15,891	7,392	8,064	9,429	11,421	17,661
150	17,325	7,392	8,064	9,429	11,421	17,661
150	17,325	7,392	8,064	9,429	11,421	17,661
150	17,661	7,392	8,064	9,429	11,421	17,661
150	17,661	7,392	8,064	9,429	11,421	17,661
150	17,661	7,392	8,064	9,429	11,421	17,661

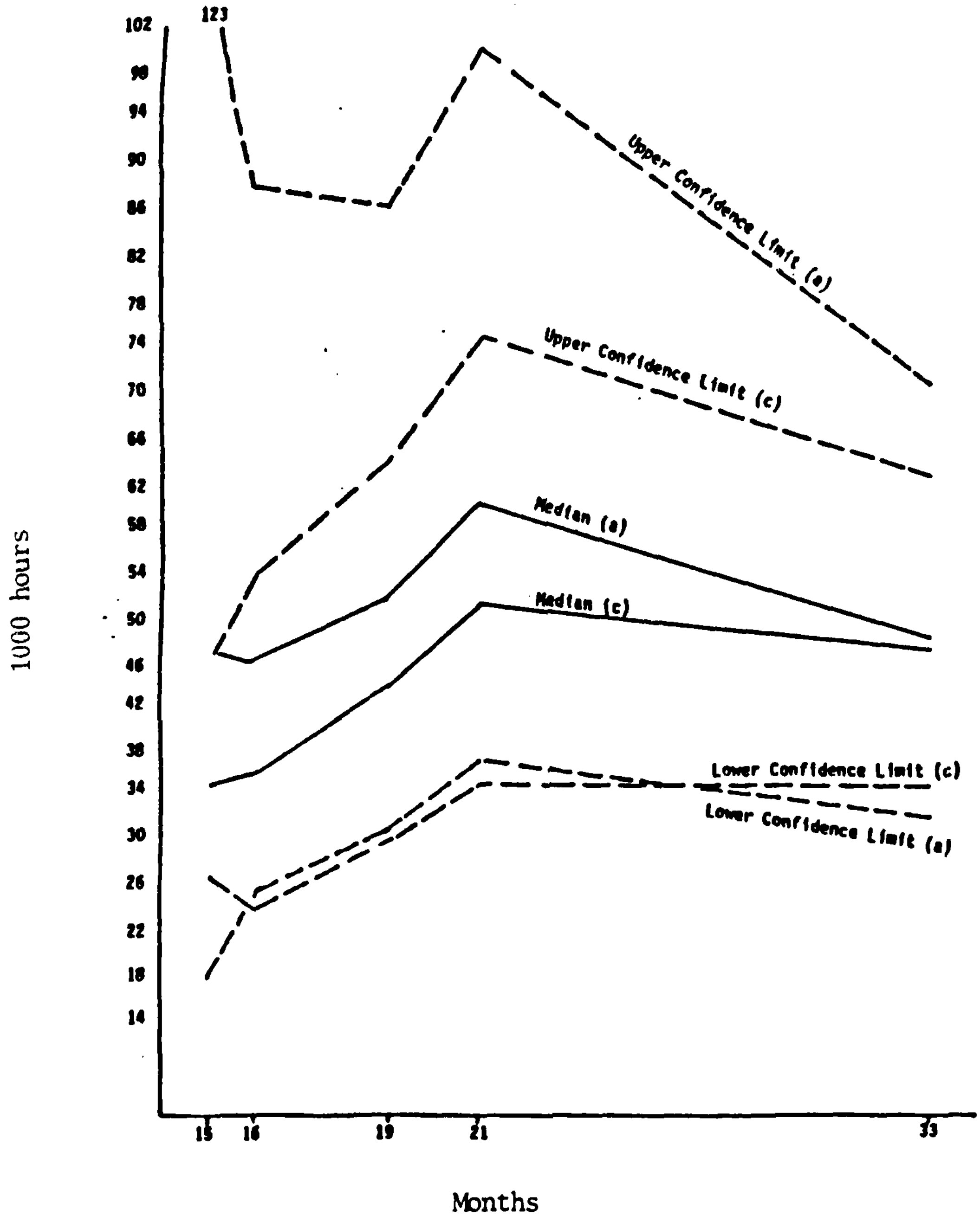

Figure 2 Median life in 1,000 hours and 95% confidence intervals, (a) with all data, (c) the two earliest observations at 190°C censored.

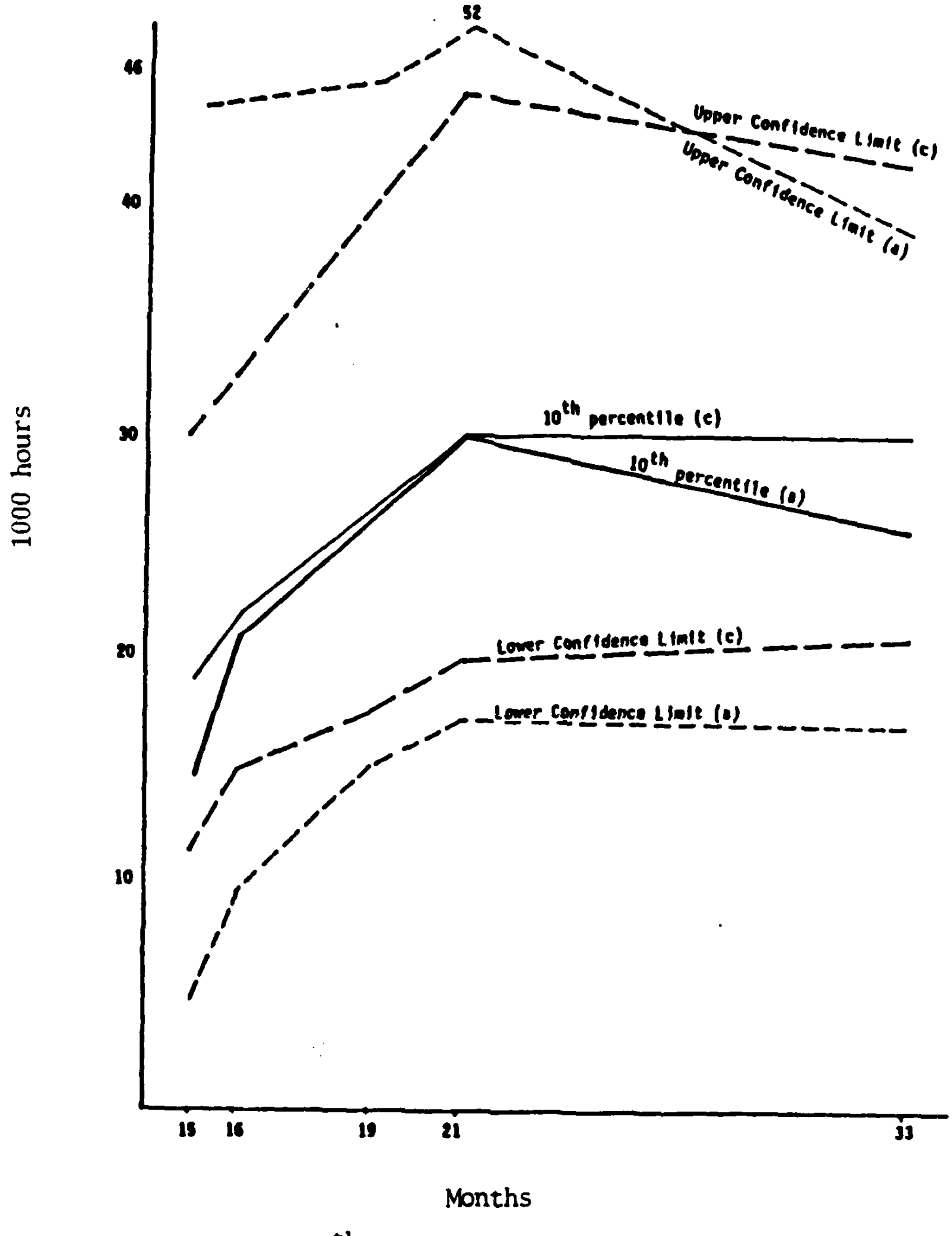

Figure 3 10th percentile life in 1,000 hours and 95% confidence intervals, (a) with all data, (c) the two earliest observations at 190°C censored.

END

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